Communication for maths

Term 2 week 7 - Integration: On areas and volumes.

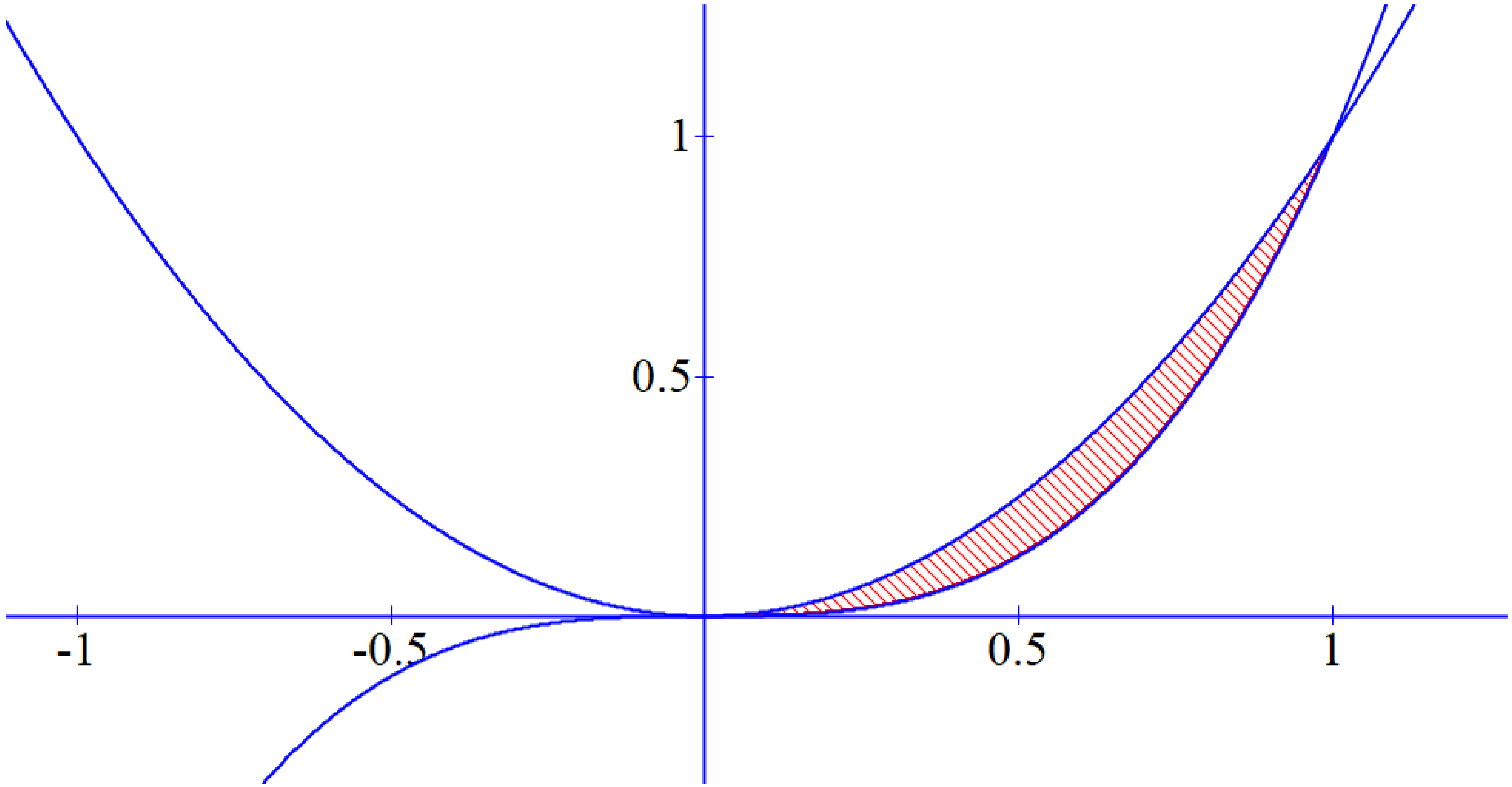
Introduction

- These slides illustrate certain aspects relating to the presentation and discussion of areas under or between curves.
- Some relate to the presentation of all maths in general and some are specific to the presentation of areas.
- Not all aspects of mathematics communication that we have studied so far will be presented here.
- Revise the slides on the previous topics where appropriate.

Mathematical presentation of area problems

Present limit calculations

Find the area bounded between $y_1 = x^2$ and $y_2 = x^3$.



Mathematical presentation of area problems

Present limit calculations

Solution (Incorrect. Why ?)

$$A = \int_0^1 x^2 - x^3 \, dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} .$$

Mathematical presentation of area problems

Present limit calculations

Solution (Correct)

See lesson

Mathematical presentation of area problems

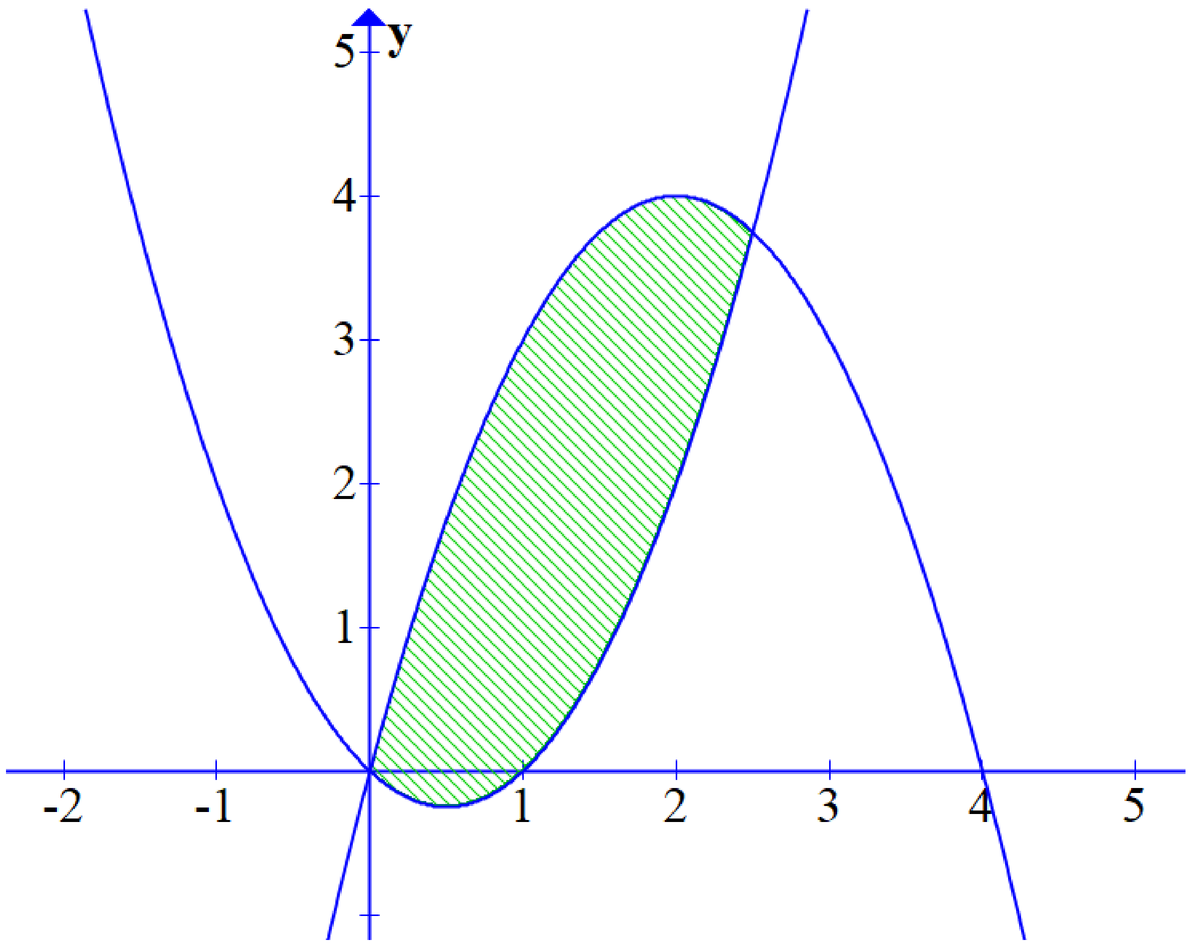
Present "upper – lower" function correctly

Find the area

bounded between

$y_1 = x(x - 1)$ and

$y_2 = x(x - 4)$



Mathematical presentation of area problems

Present "upper – lower" function correctly

Solution (incorrect)

Given $y_1 = x^2 - x$ and $y_2 = -x^2 + 4x$, points of intersection of the curves are found as follows:

$$x^2 - x = -x^2 + 4x \Rightarrow x(2x - 5) = 0$$

$$\therefore \; x = 0, \; 5/2$$

Hence $A = \int_0^{5/2} -x^2 + 4x - (x^2 - x)\,dx = \ldots$ (left as Ex)

Mathematical presentation of area problems

Present “upper – lower” function correctly

Solution (correct)

See lesson

Ways of speaking

- We saw in our work on "curve sketching – transformations" that the following description is just a verbalisation of the symbols of $y = mx + c$

"y equals m times x plus c"

- An actual conceptual description of $y = mx + c$ is

"This is a straight line of gradient m, y-intercept c and x-intercept –c/m."

Describing integral representation of areas

Exercises

- Bearing in mind the previous slide, describe in plain English, and with all relevant detail and precision, what the following integrals represent:

1) $$\int_a^a f(x)\ dx$$

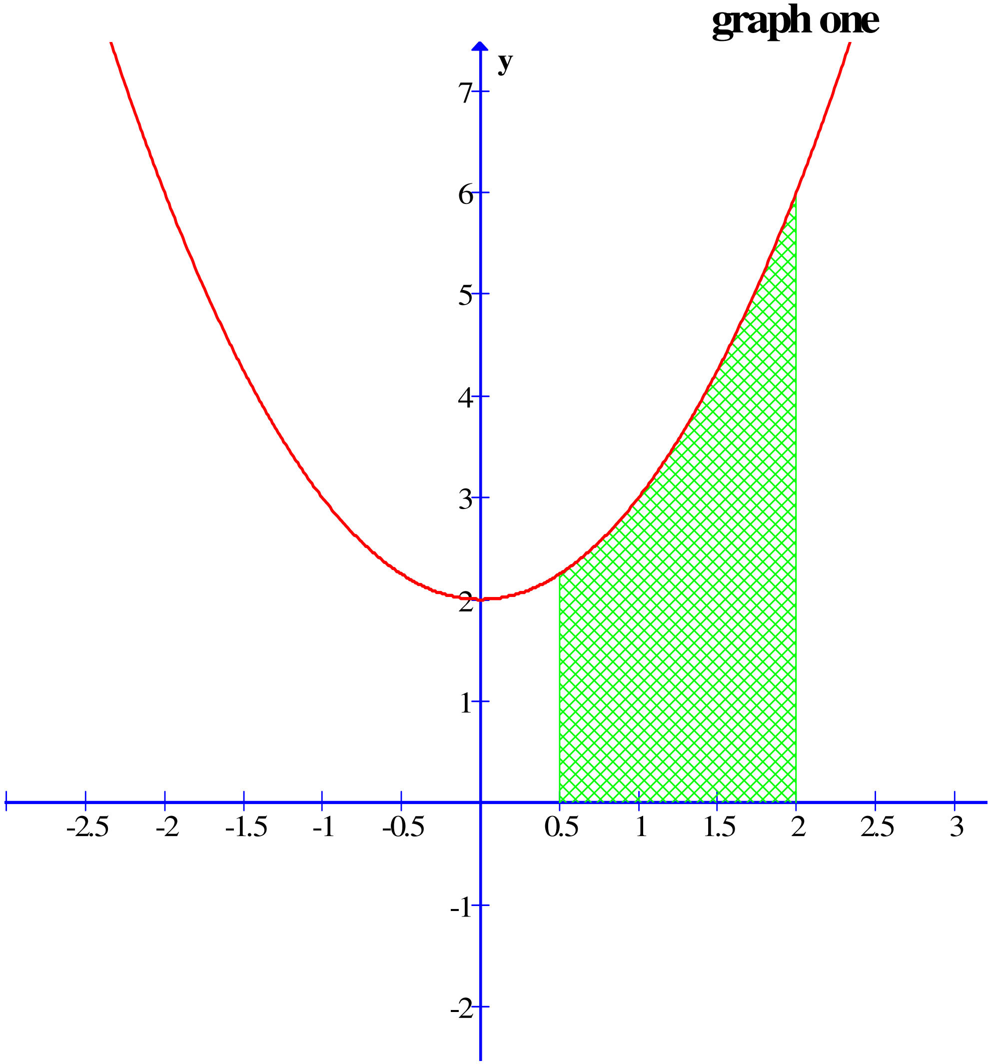
2) $$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$

3) $$\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$$

Describing integral representation of areas

Exercise 1

Identify relevant vocabulary for describing the area highlighted in green, where $y = x^2$.



Describing integral representation of areas

Exercise 1

Using this vocabulary

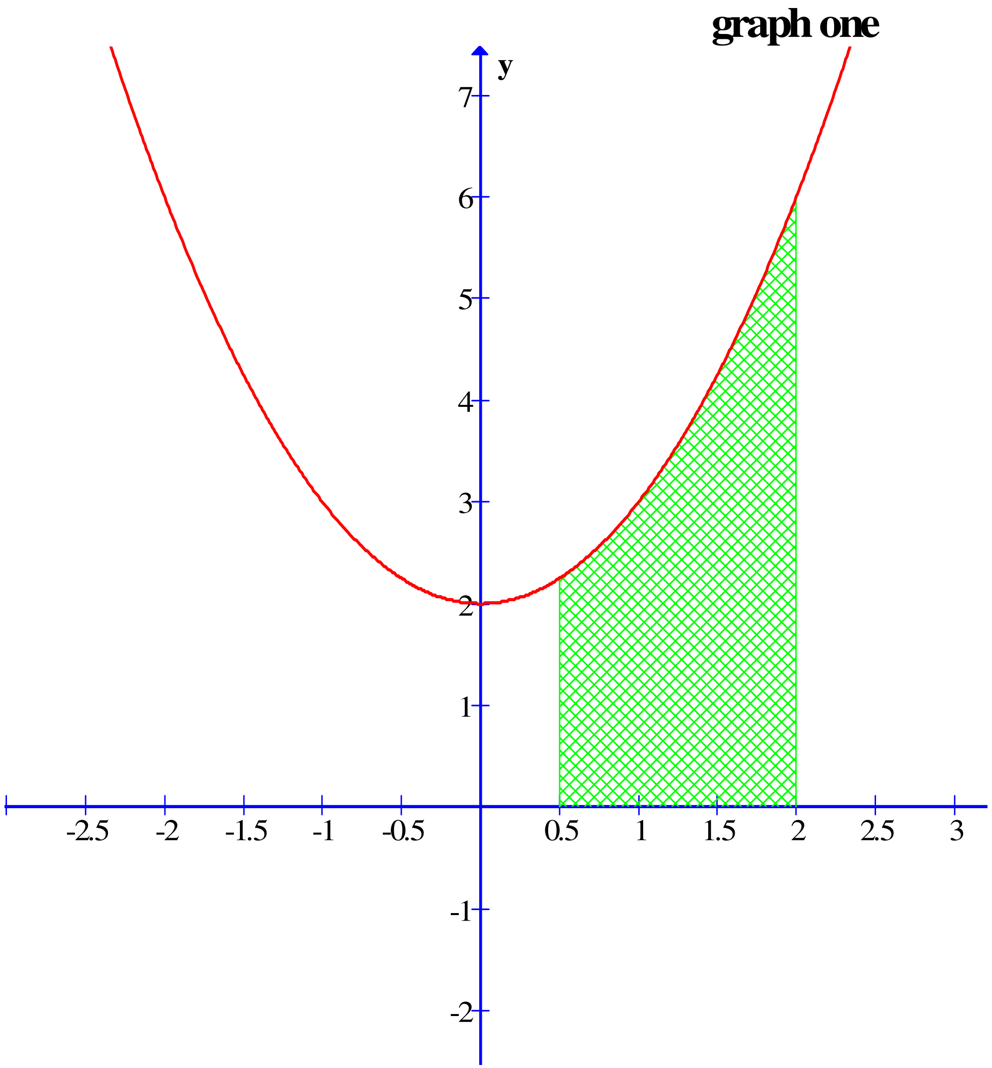
describe in plain English

the process for finding

the area highlighted

in green,

where $y = x^2$.



Describing integral representation of areas

Exercise 2

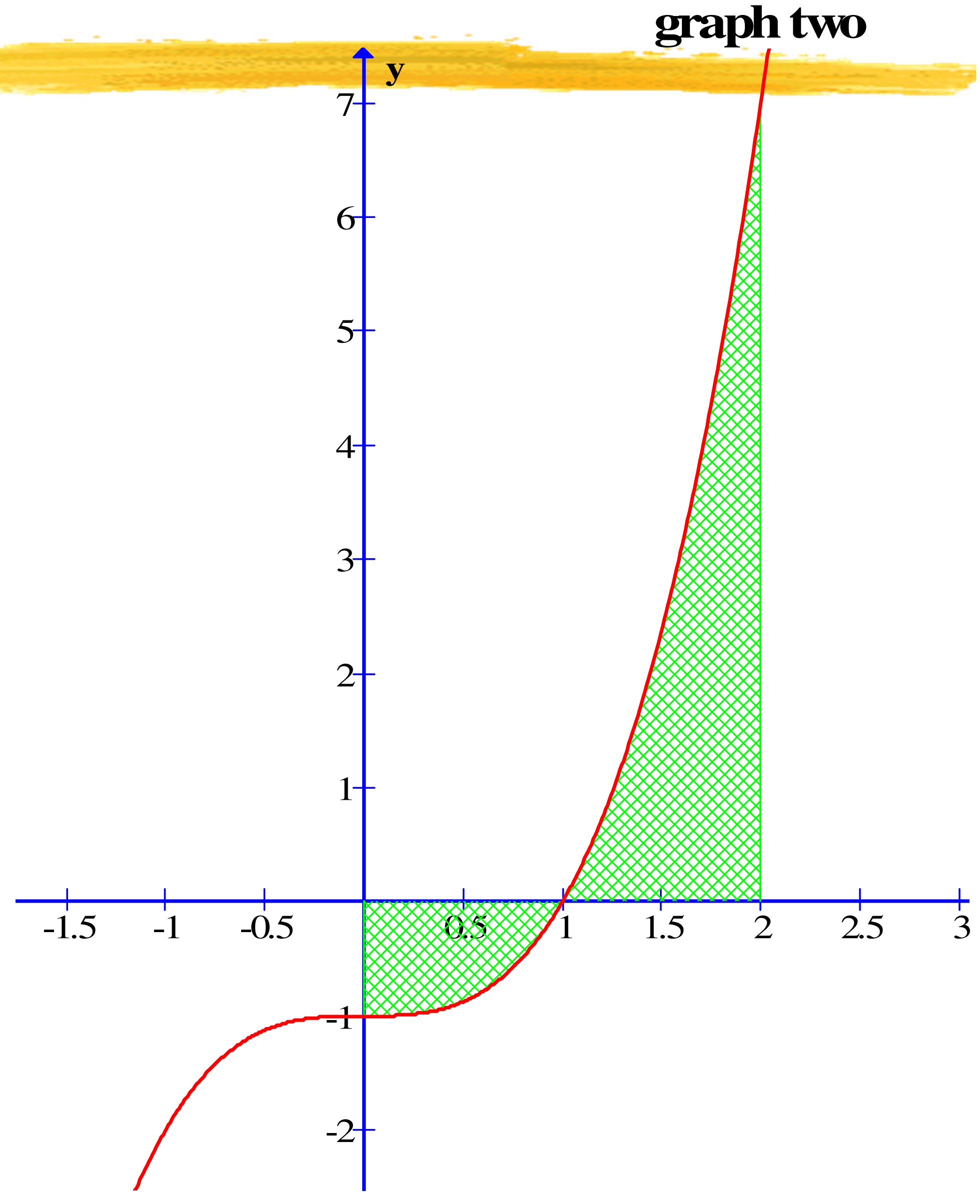
In the same way,

describe the process

for finding the area

highlighted here,

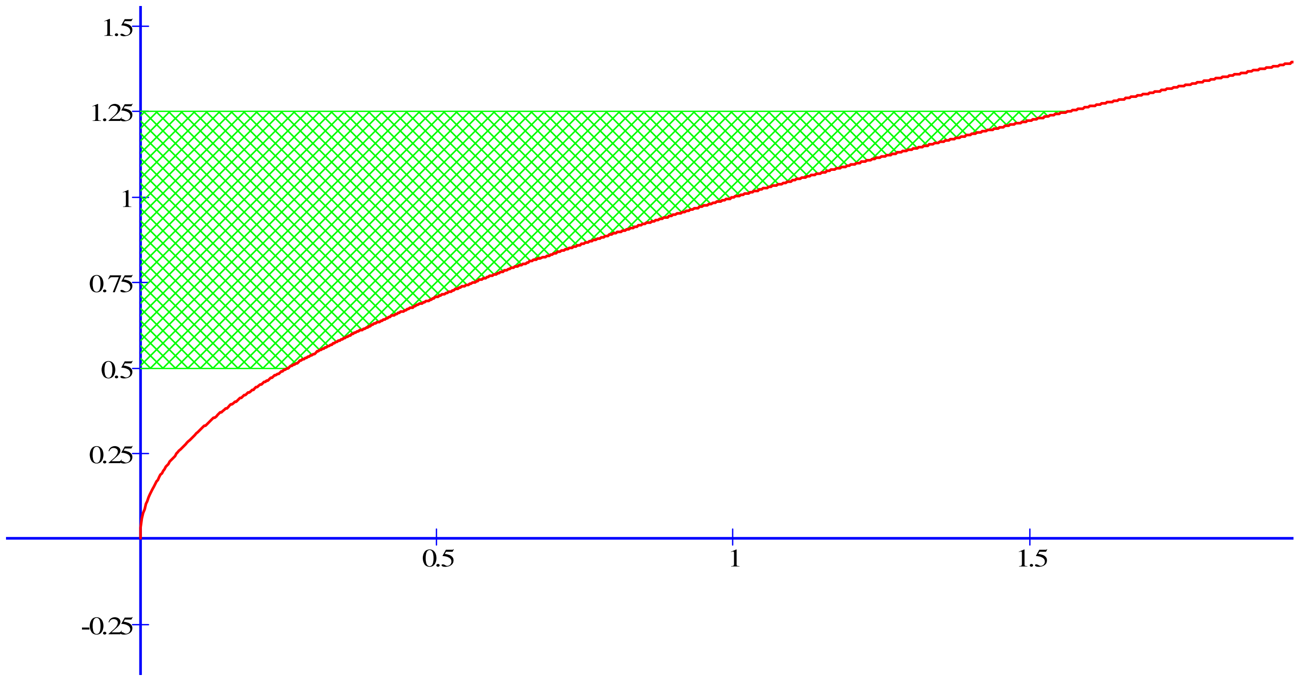
where $y = x^3 - 1$.



Describing integral representation of areas

Exercise 3

In the same way, describe the process for finding the area highlighted here, where $y^2 = x$.



Describing integral representation of areas

Exercise 4

In the same way,

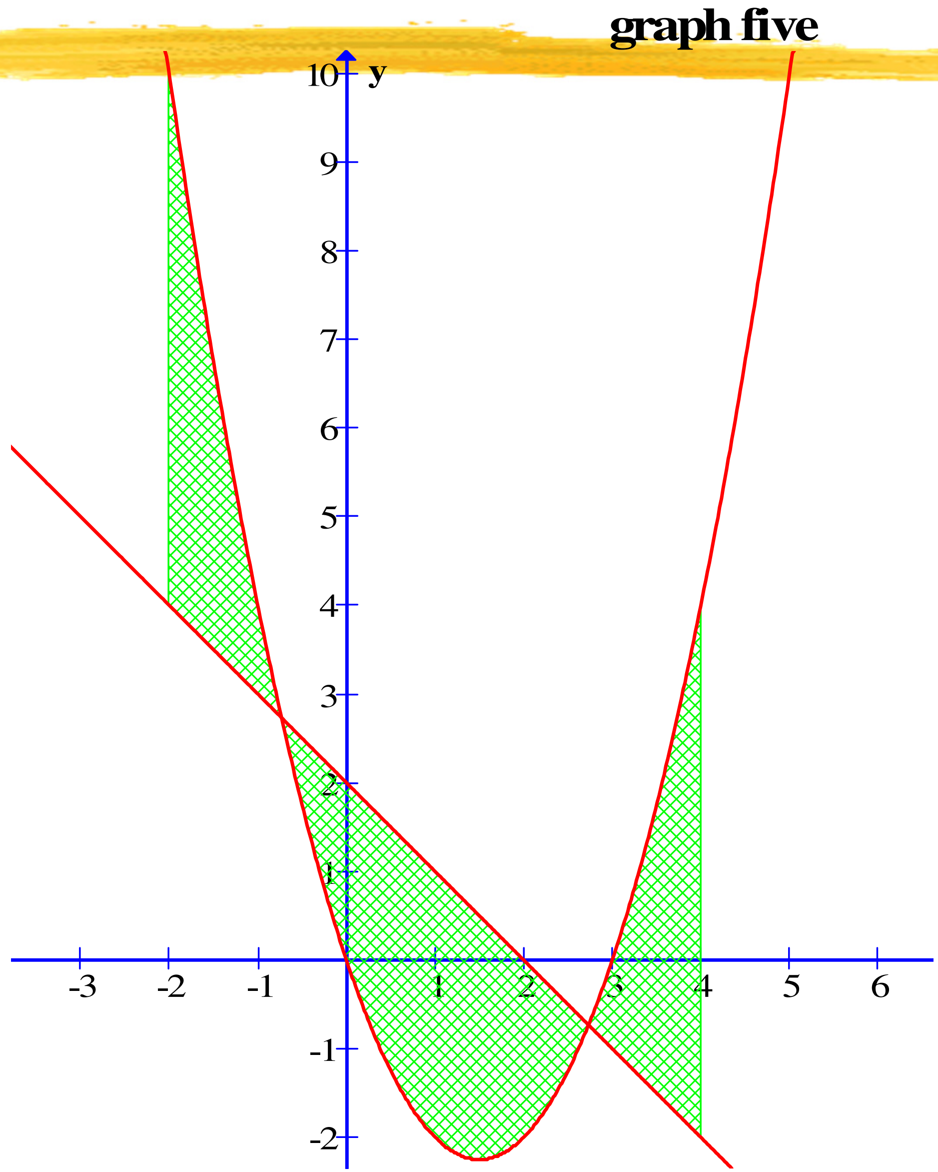
describe the process

for finding the area

highlighted here,

where $y_1 = x^2 - 3x$

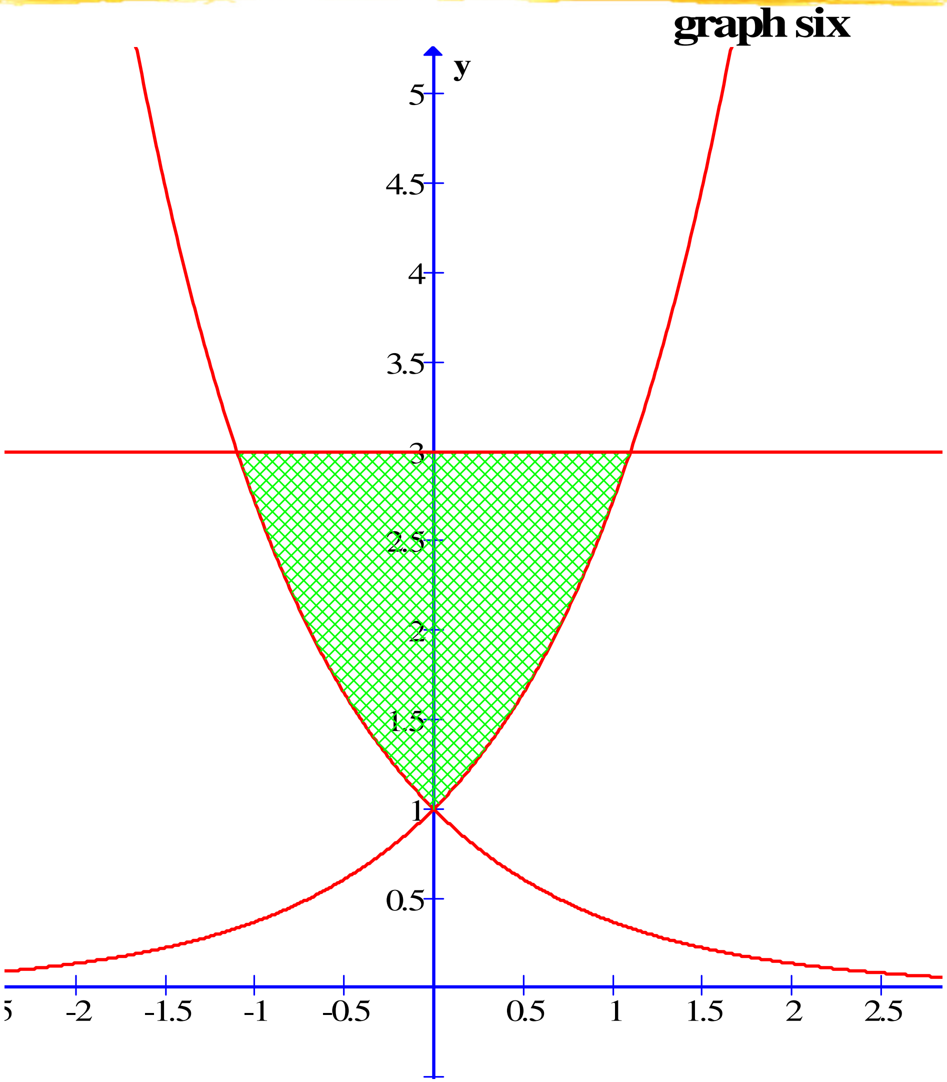
and $y_2 = -x + 2$.



Describing integral representation of areas

Exercise 5

In the same way, describe the process for finding the area highlighted here, where $y_1 = e^x$ and $y_2 = e^{-x}$ and $y_3 = 3$.



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Answers/solutions

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Limit calculations example: Correct solution

<u>Limits</u>: Solve $x^2 = x^3$.

Hence $x^3 - x^2 = x^2(x-1) = 0 \Rightarrow x = 0, 1$

Now, for any $x \in [0,1]$, $x^2 > x^3$, Therefore

$$A = \int_0^1 x^2 - x^3 \, dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$$

"upper – lower" function example: Correct solution

given $y_1 = x^2 - x$ and $y_2 = -x^2 + 4x$, Points of intersection of the curves are as follows:

$$x^2 - x = -x^2 + 4x \Rightarrow 2x^2 - 5x = 0 \Rightarrow x(2x-5) = 0$$

$\therefore$ $x = 0, 5/2$. Now let $x = 2$. Then $y_1(2) = 3$ and $y_2(2) = 4$.

$\therefore$ For all $x \in [0, 5/2]$, $y_2 > y_1$.

Hence

$$A = \int_0^{5/2} y_2 - y_1 \, dx = \int_0^{5/2} -x^2 + 4x - (x^2 - x) \, dx$$

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Appendix

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Describing integral representation of areas

Example

… can be describe in English as

To find the area bounded by the curve

and the x-axis between x = 0.5 and x = 2,

we integrate $y = x^2$ between x = 0.5 and x = 2.